Splines

Bézier curve

- Invented by French engineer, Pierre Bézier (1910-1999).
- Advantages:
  – Draw a smooth curve through a series of linear interpolations.

Bézier curve: Application

- Where could Bézier curves be used?
  - To draw smooth, vector-based typefaces like Microsoft TrueType or Adobe Postscript.
  - In computer aided design (CAD)
  - In paint and draw programs

n degree Bézier curve

$$Bezier\,(n,t) = \sum_{i=0}^{n} \binom{n}{i} (1-t)^{n-i} t^{i}\, w^{i}$$

Example Bézier curve

$$Bezier\ (n,t) = \sum_{i=0}^{n} \binom{n}{i} (1-t)^{n-i} t^{i}\, w^{i}$$

P1: (120, 160) ; P2: (35, 200); P3: (220, 260); P4: (220, 240); n = 3

$x = (1-t)^{(3-0)} * t^{0} * 120 + (1-t)^{(3-1)} * t^{1} * 3 * 35 + (1-t)^{(3-2)} * t^{2} * 3 * 220 + (1-t)^{(3-3)} * t^{3} * 220$

$y = (1-t)^{(3-0)} * t^{0} * 160 + (1-t)^{(3-1)} * t^{1} * 3 * 200 + (1-t)^{(3-2)} * t^{2} * 3 * 260 + (1-t)^{(3-3)} * t^{3} * 240$

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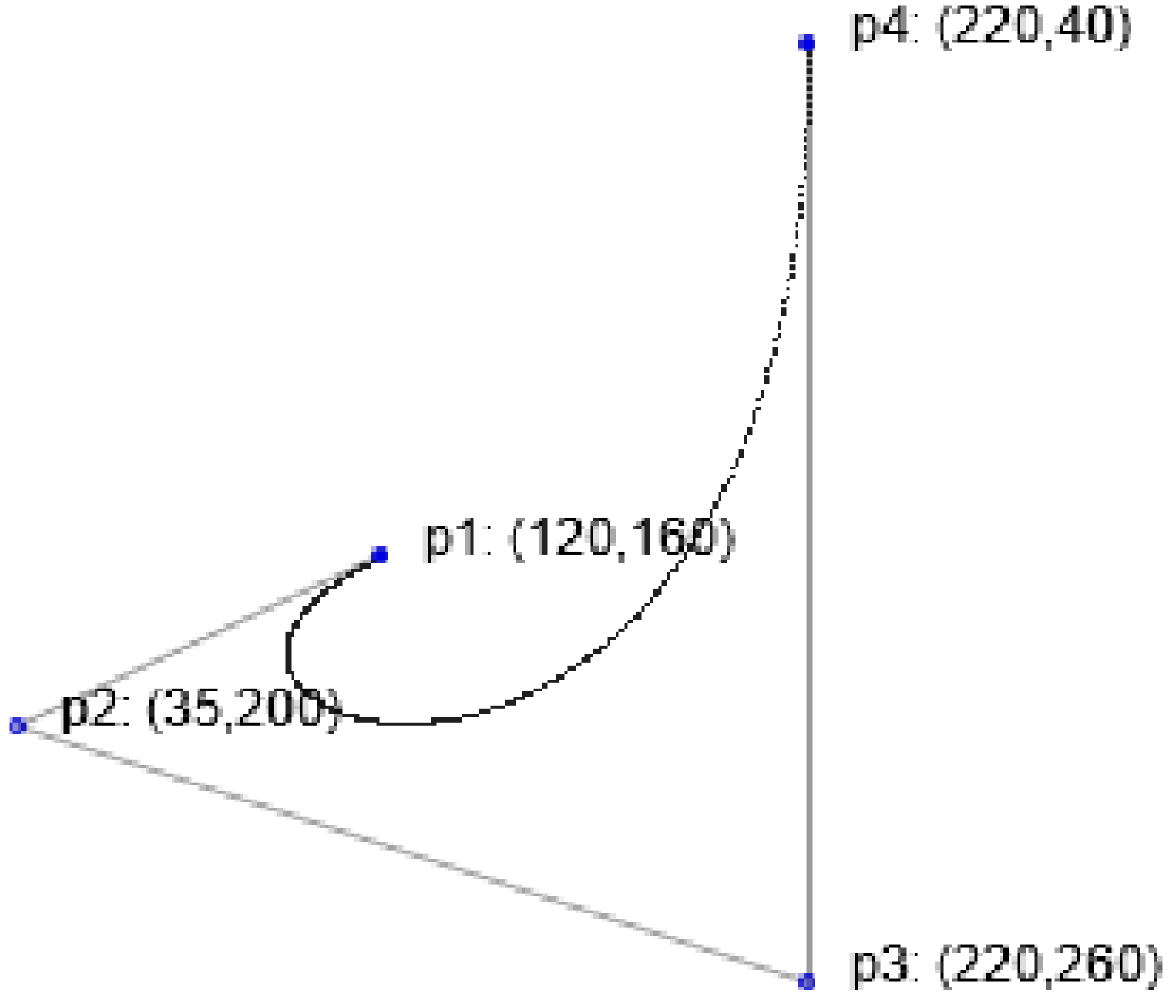
Solve for values of t between 0 and 1 inclusive.

t = 0: x = 120; y = 160
t = 0.25: x = ?; y = ?
t = 0.5: x = ?; y = ?
t = 1: x = ?; y = ?

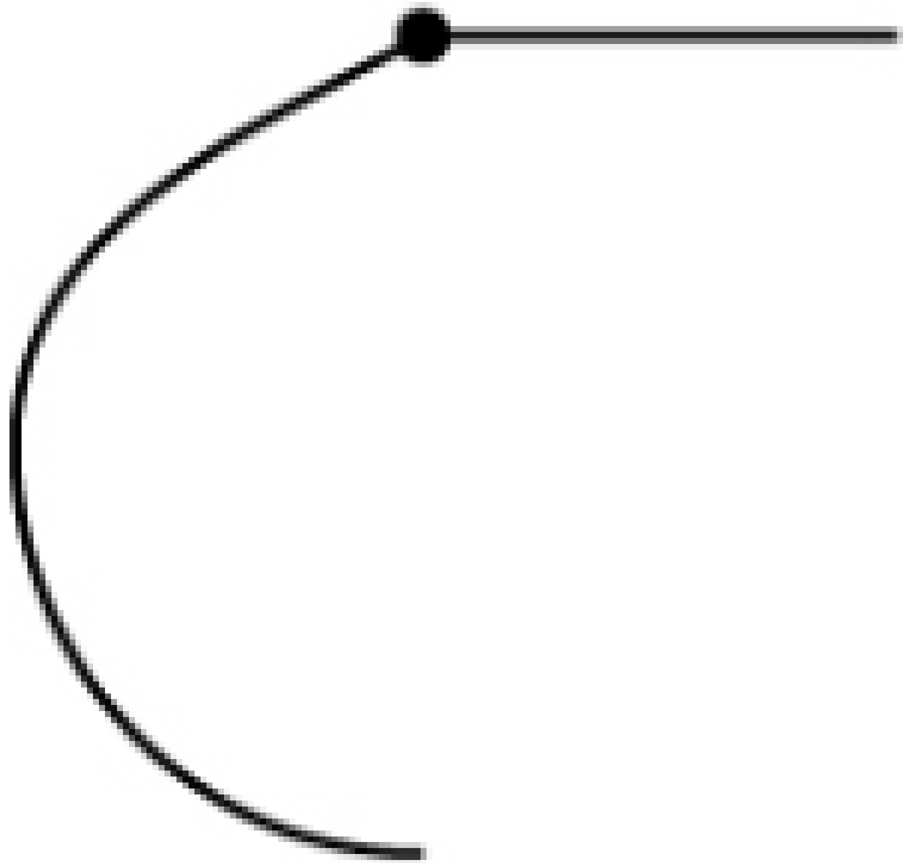
Example Bézier curve



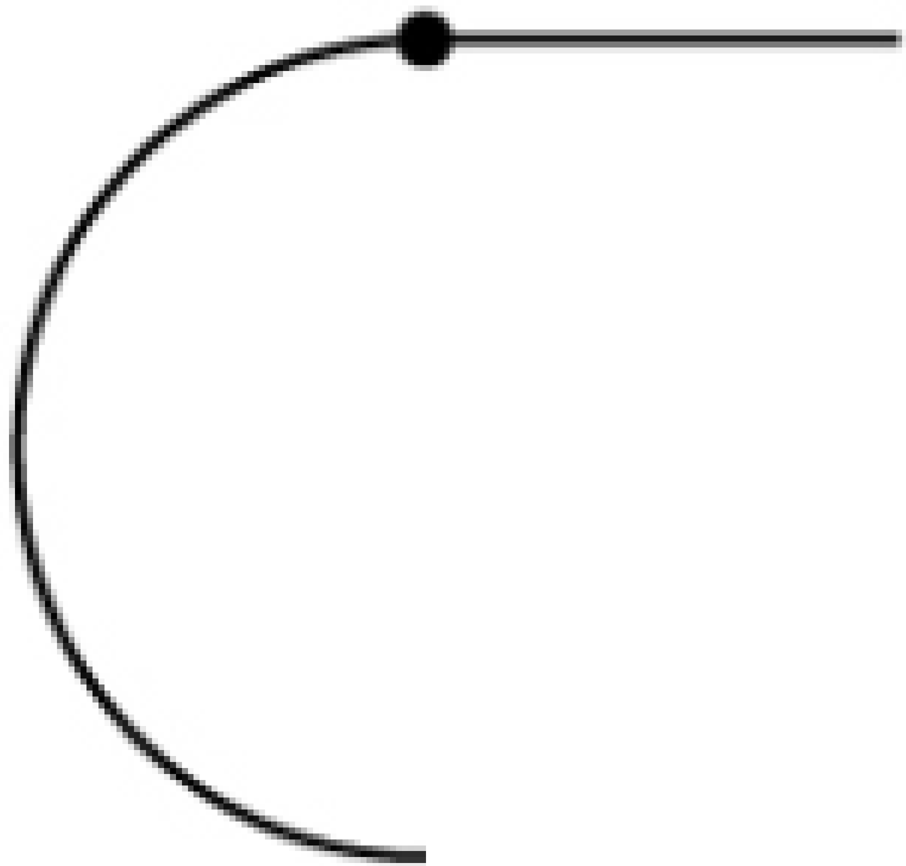
Continuity

- Two Bézier curves can be joined together.
- Smoothness of curve at juncture point determined by continuity: C^0, C^1, and C^2.
- C^0 continuity:
  - curves are joined together and share a common point.
- C^1 continuity:
  - First derivative at juncture point of both curves is equal.
- C^2 continuity:
  - First and second derivative at juncture point of both curves is equal.

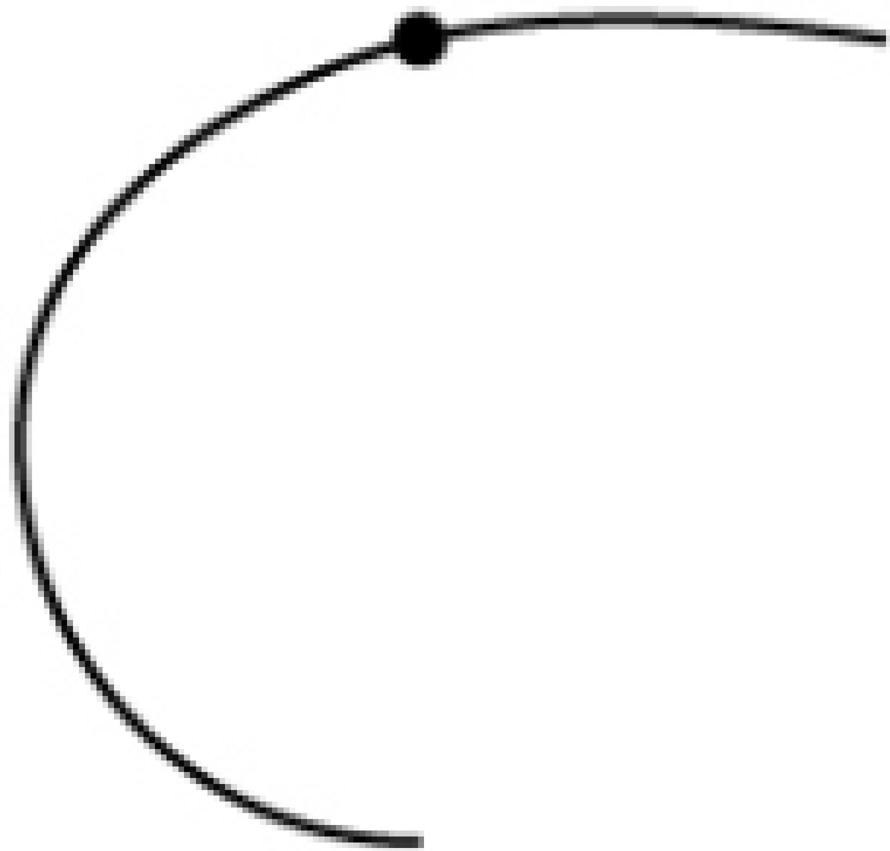
C^0 continuity



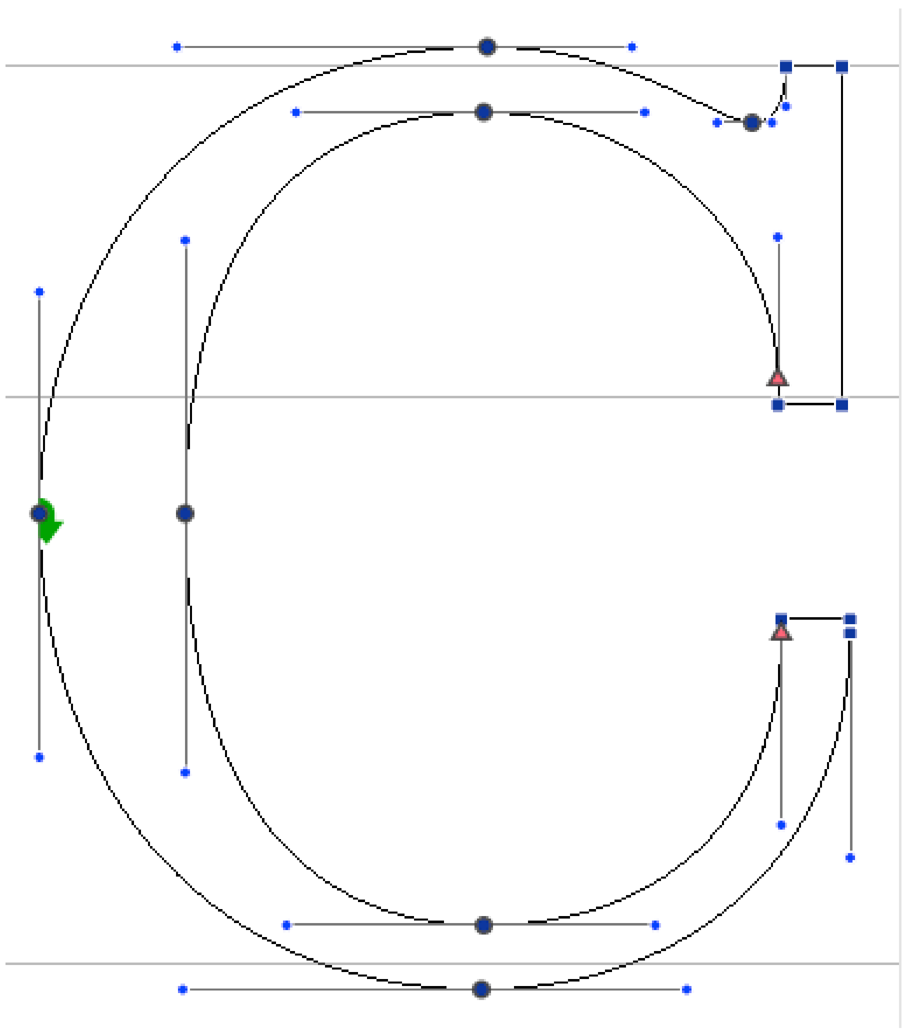
C^1 continuity



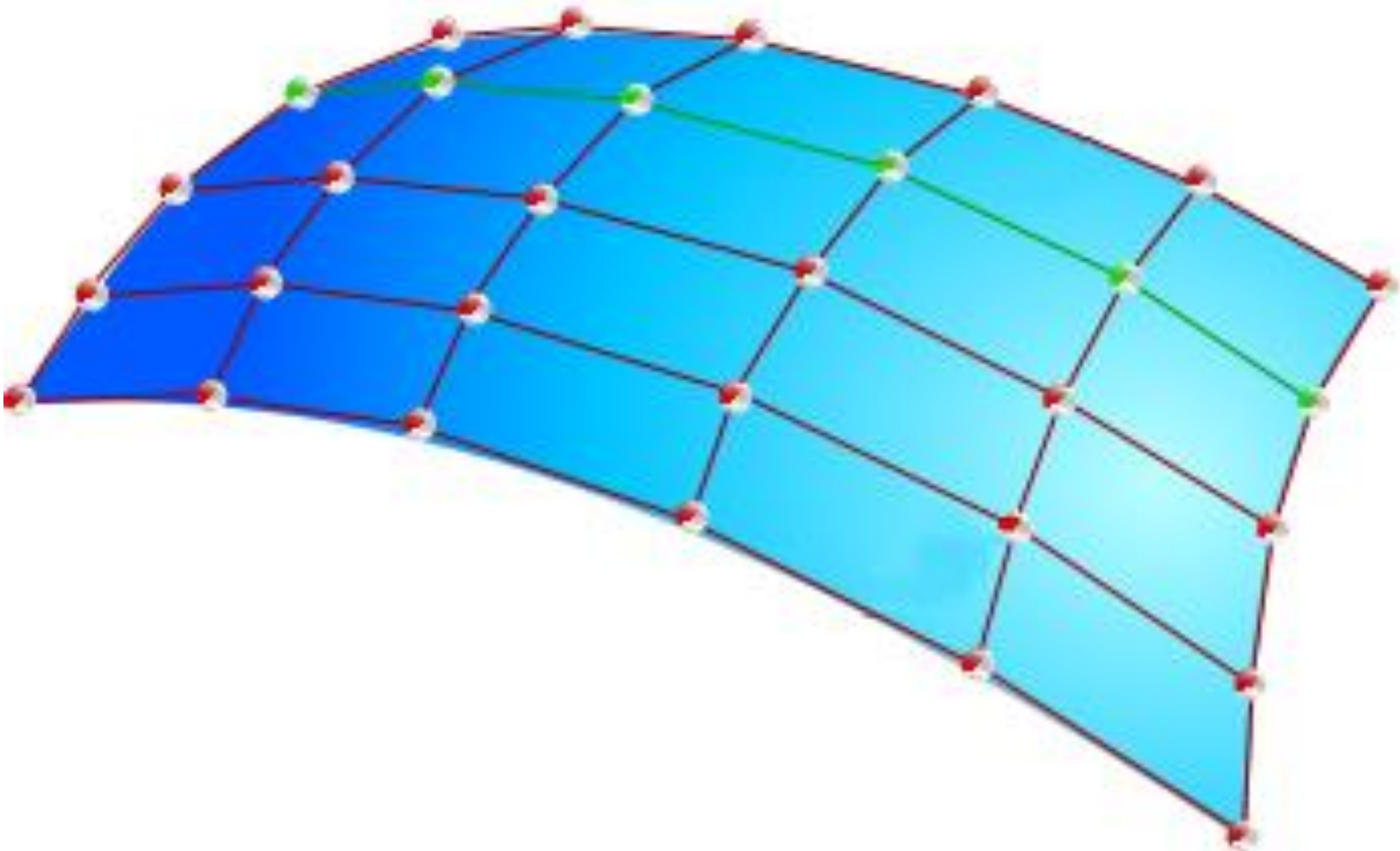
C^2 continuity



Bézier curve application: typography



Meshes



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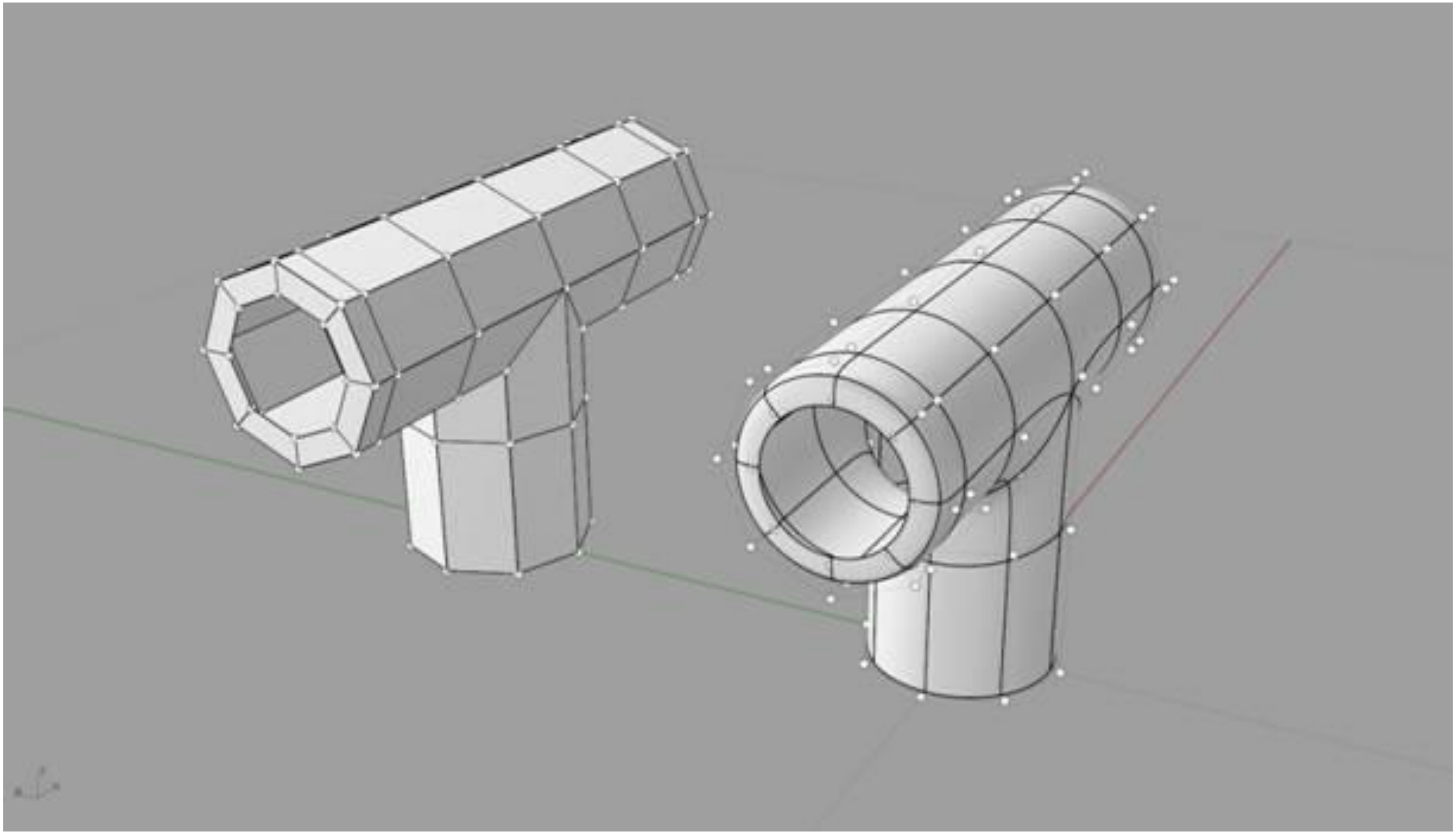

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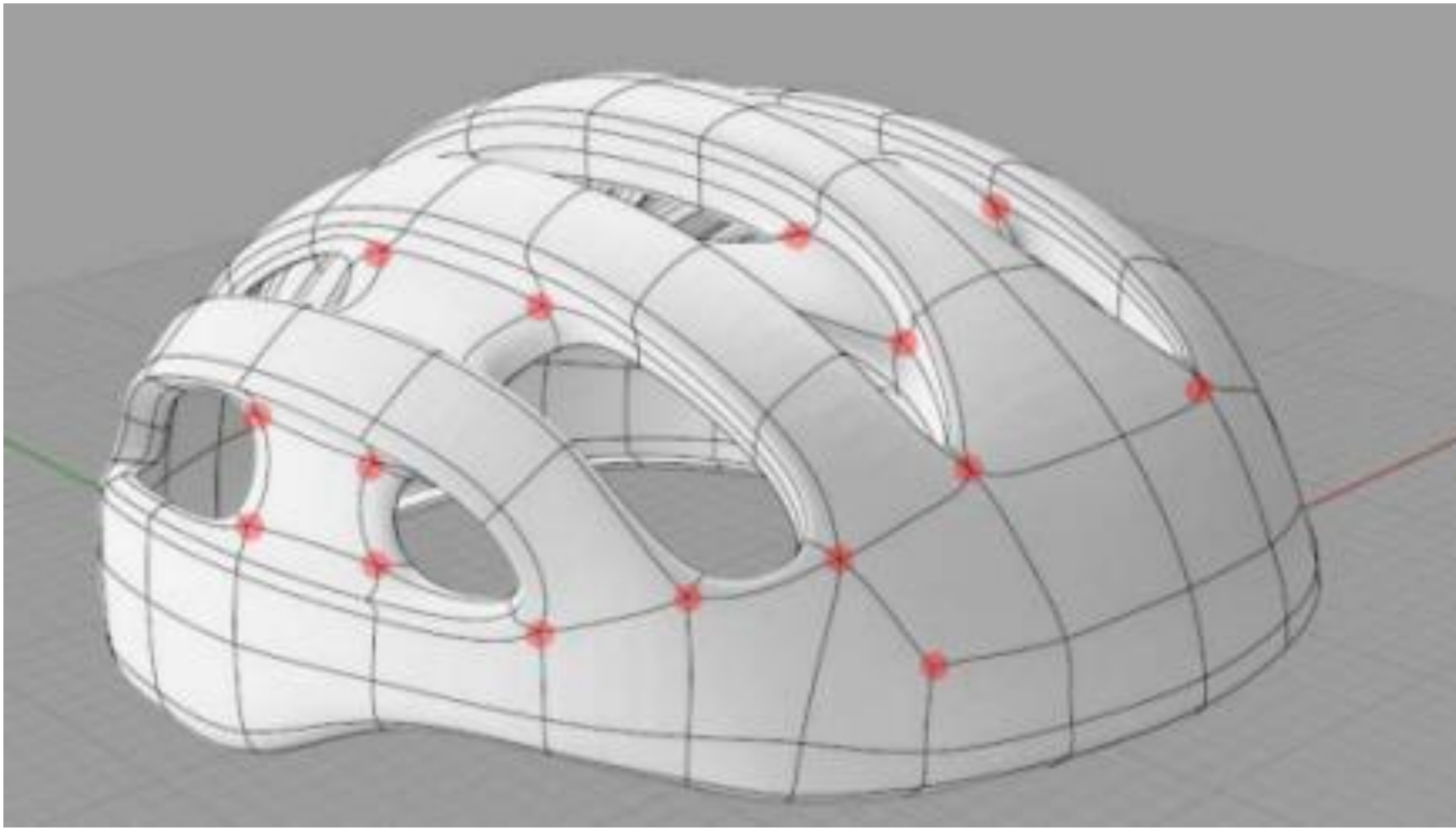

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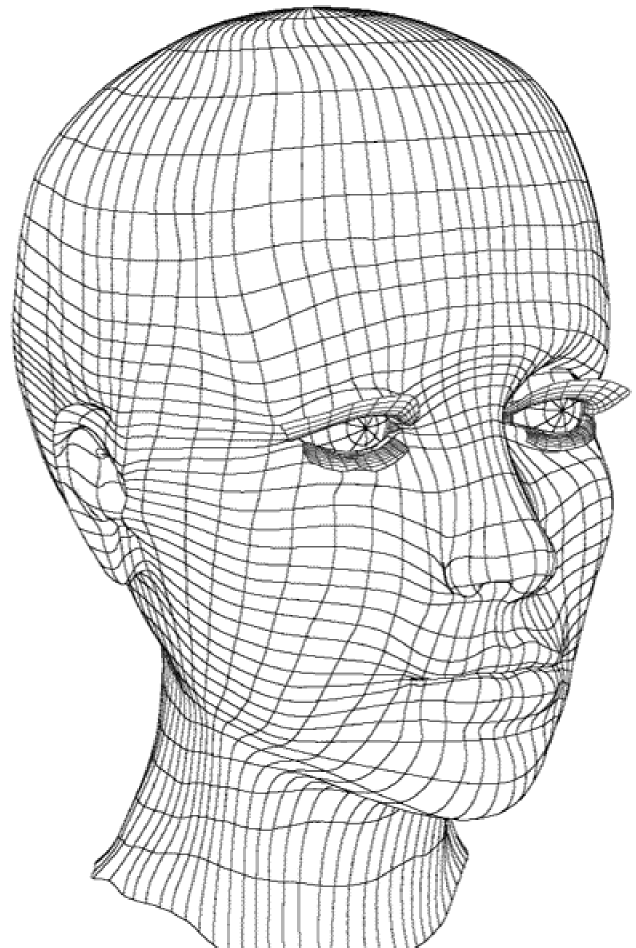
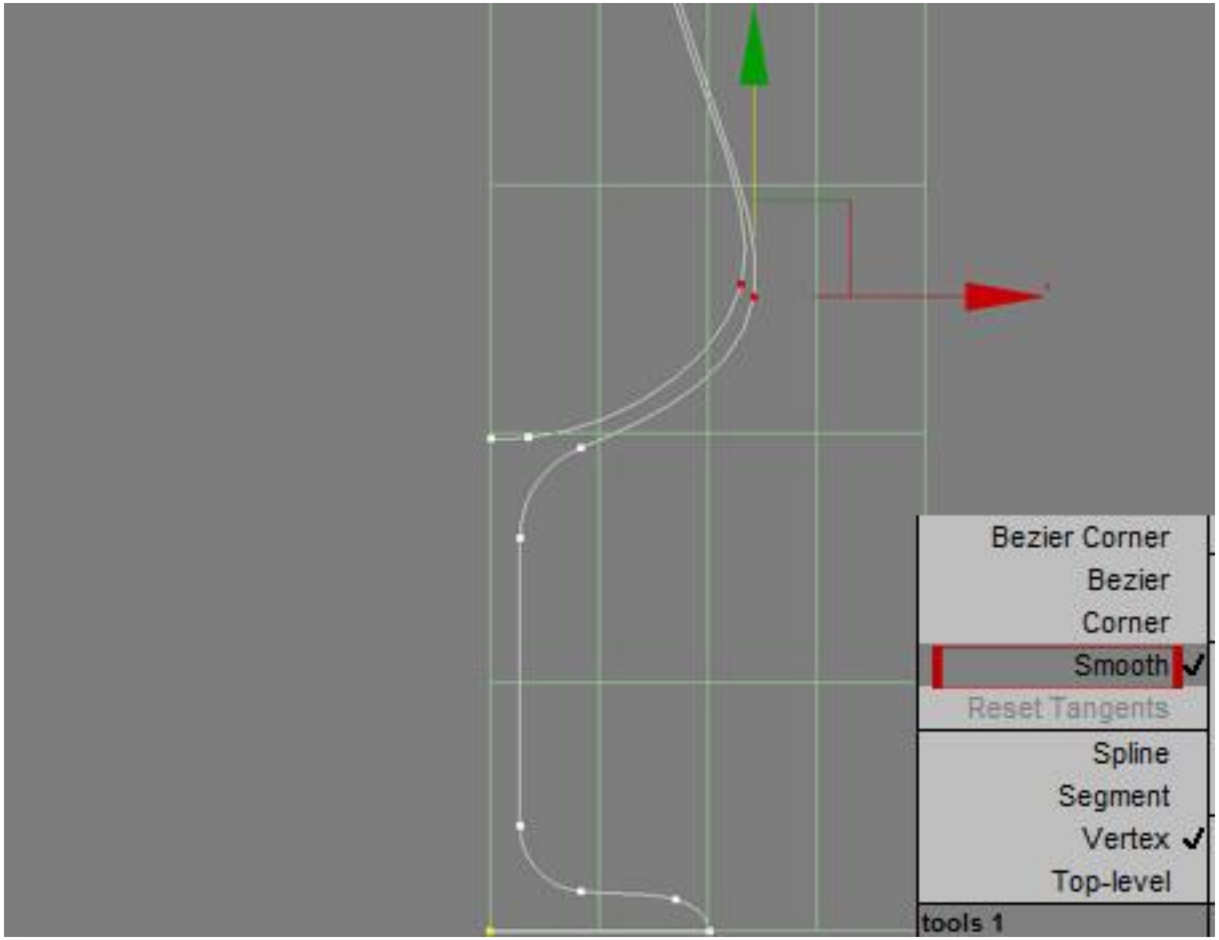


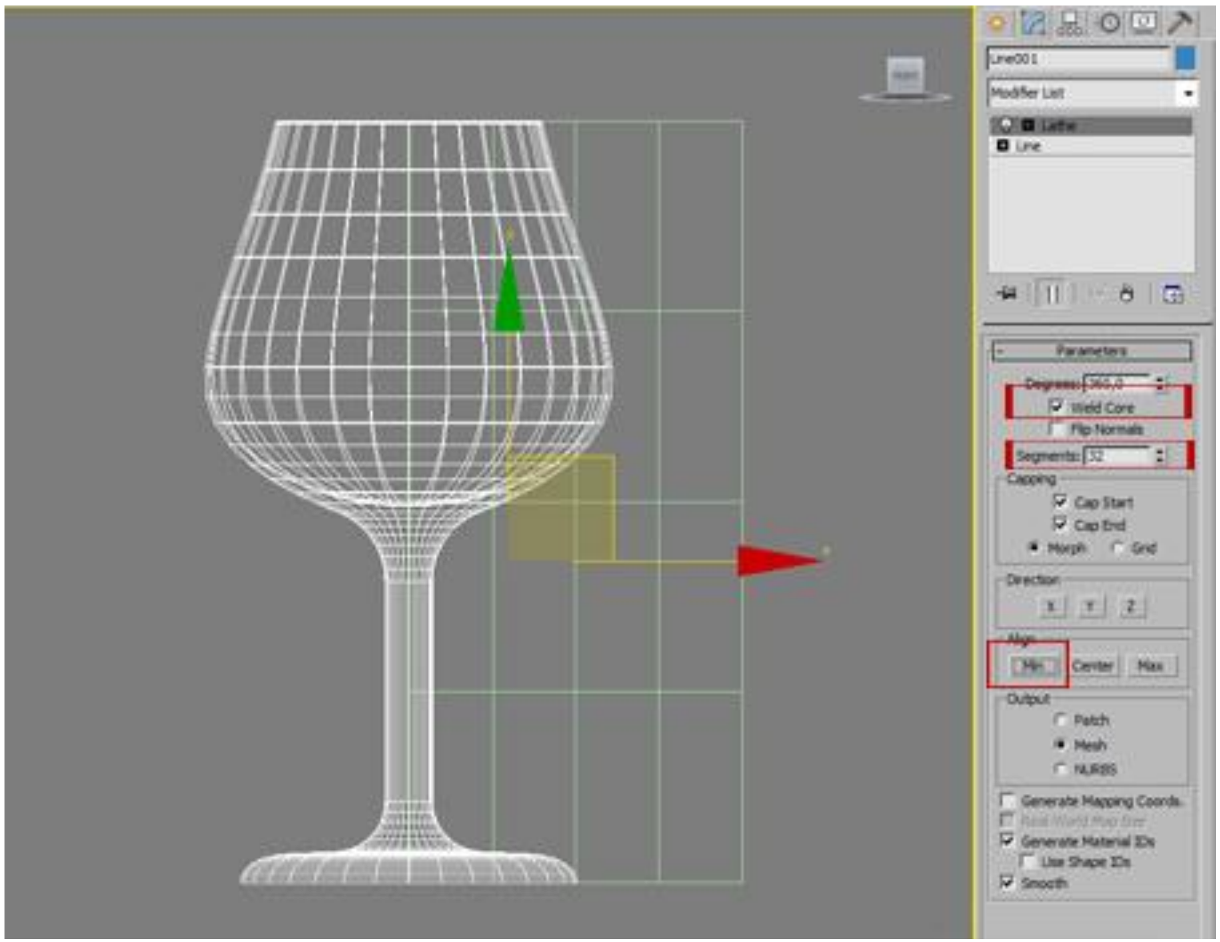
Lathe functionality

- A method for rapidly creating complex 3D objects.
  - Start with a 2D curve.
  - Spin the curve around an axis (X, Y, or Z)
  - Instant volume-based 3-dimensional object!

Lathe function



Lathe function



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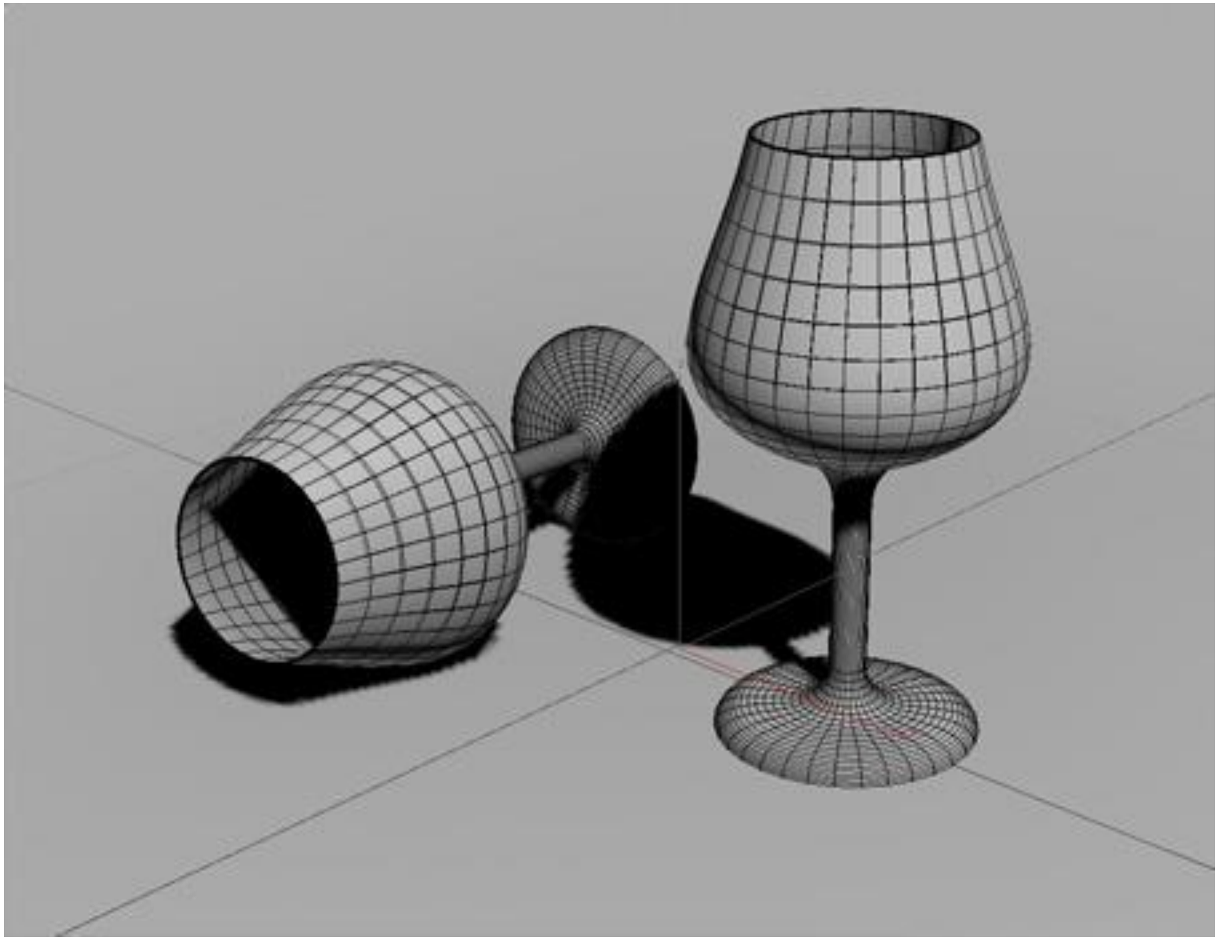

For further reading…

An introduction to splines for use in computer graphics and geometric modeling by Richard H. Bartels, John C. Beatty, and Brian A. Barsky

Curves and surfaces for computer aided geometric design: A practical guide by Gerald Farin

Curves and Surfaces for Computer Graphics by David Salomon

A practical guide to splines by Carl de Boor

The NURBS Book by Les A. Piegl