A review of classical mechanics in physics

CS-116B Graphics Algorithms

Newton's three laws of motion

- **1st law of motion (law of inertia)**
  - A body remains at rest, or moves in a straight line (at a constant velocity), unless acted upon by a net outside force.
- **2nd law of motion (F = m * a)**
  - The acceleration of an object is proportional to the force acting upon it.
- **3rd law of motion (law of reciprocal actions)**
  - For every action, there is an equal and opposite reaction.

The Newton: a unit of force

- The Newton, N, is expressed as:

  $N = kg * m / s^2$

Distance, velocity, and acceleration

- Distance signified by ***s***
- Velocity, ***v***, is the change in distance over time:

  $v = ds/dt$
- Acceleration, ***a***, is the change in velocity over time:

  $a = dv/dt$

  $= d^2s/dt^2$

Exploding cubes: how to displace particles?

- When working on the exploding cube programming assignment, consider the following:

Displacement of a particle under constant acceleration

$s = v_0 t * ½ at^2$

v_0 is the initial velocity at time t

Rope and cloth simulations: forces we will use

- We will be using Newton’s second law (F = m * a) and Hooke’s law (F = k * x) to compute the displacement of particles for our rope and cloth simulations.

Vectors

A vector contains two components:

- A scaler value
- A direction

Example: vector v = Aî + Bĵ + Ck̂

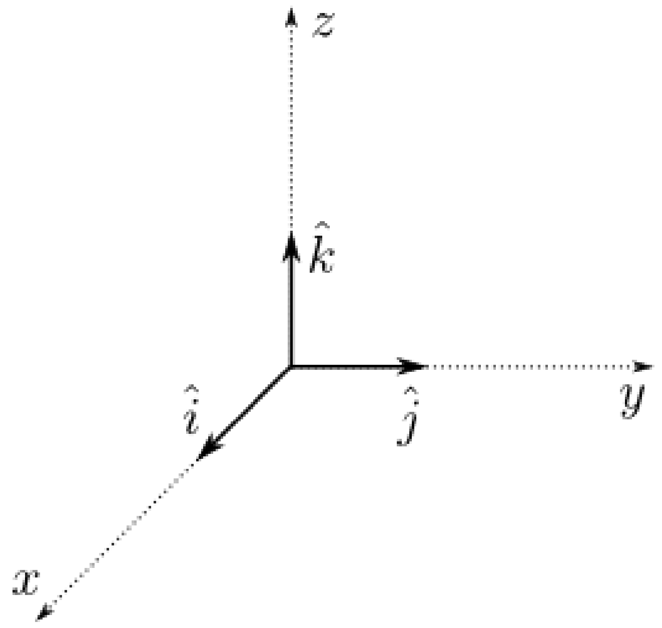


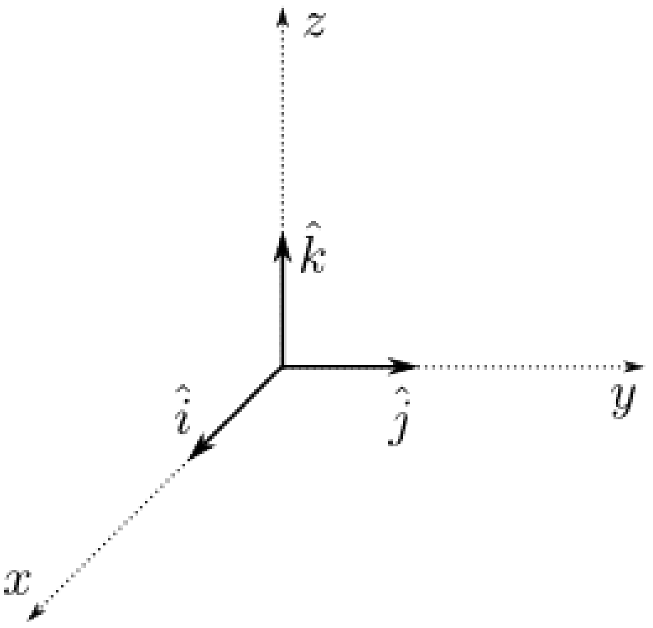
Image source: https://upload.wikimedia.org/wikibooks/en/9/98/Unit-vectors-in-Cartesian-Coord.png

Computing vector magnitude

vector v = Aî + Bĵ + Ck̂

Magnitude of vector v:

$||v|| = SQRT(A^2 + B^2 + C^2)$



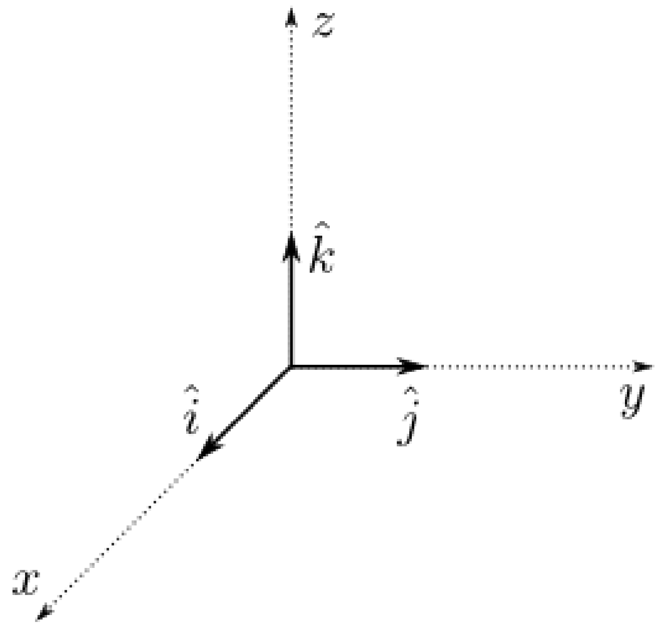
Normalizing a vector
(also called unit vector)

vector v = Aî + Bĵ + Ck̂

normalized vector v

= v / ||v||

= **v / SQRT(A^2 + B^2 + C^2)**



Computing the vector dot product

(also called scalar product)

vector $v_1 = A\hat{i} + B\hat{j} + C\hat{k}$

vector $v_2 = D\hat{i} + E\hat{j} + F\hat{k}$

$v1 \cdot v2 = ||v_1|| * ||v_2|| * \cos(\theta)$

$= A\hat{i} \cdot D\hat{i} + A\hat{i} \cdot E\hat{j} + A\hat{i} \cdot F\hat{k} + B\hat{j} \cdot D\hat{i} + B\hat{j} \cdot E\hat{j} + B\hat{j} \cdot F\hat{k} + C\hat{k} \cdot D\hat{i} + C\hat{k} \cdot E\hat{j} + C\hat{k} \cdot F\hat{k}$

$= A*D*\cos(0°) + A*E*\cos(90°) + A*F*\cos(90°) + B*D*\cos(90°) + B*E*\cos(0°) + B*F*\cos(90°) + C*D*\cos(90°) + C*E*\cos(90°) + C*F*\cos(0°)$

= **A*D + B*E + C*F**

Right hand rule

The right hand rule:

$\hat{i} \times \hat{j} = \hat{k}$

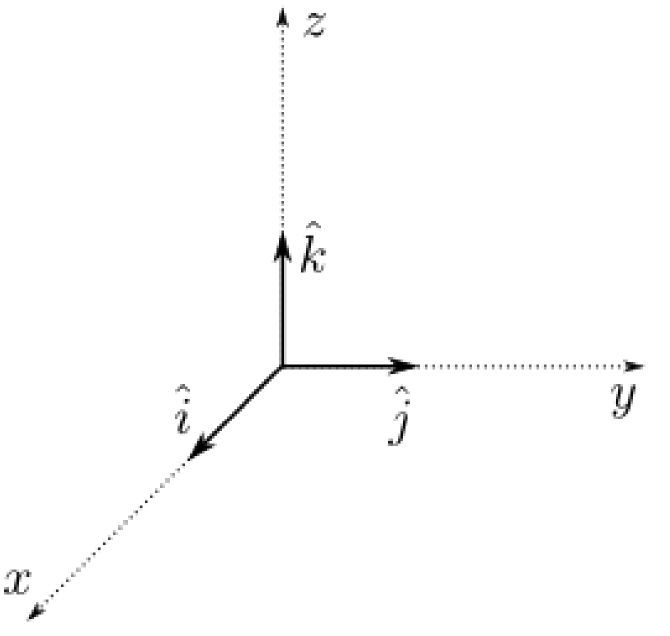
$\hat{j} \times \hat{k} = \hat{i}$

$\hat{k} \times \hat{i} = \hat{j}$

$\hat{i} \times \hat{k} = -\hat{j}$

$\hat{j} \times \hat{i} = -\hat{k}$

$\hat{k} \times \hat{j} = -\hat{i}$



Computing the vector cross product
(also called vector product)

vector $v_1 = A\hat{\mathbf{i}} + B\hat{\mathbf{j}} + C\hat{\mathbf{k}}$

vector $v_2 = D\hat{\mathbf{i}} + E\hat{\mathbf{j}} + F\hat{\mathbf{k}}$

$\hat{\mathbf{n}}$ represents the unit vector perpendicular to vectors v1 and v2 using the right hand rule.

$v_1 \times v_2 = ||v_1|| * ||v_2|| * \sin(\theta)\, \hat{\mathbf{n}}$

$= A\hat{\mathbf{i}} \times D\hat{\mathbf{i}} + A\hat{\mathbf{i}} \times E\hat{\mathbf{j}} + A\hat{\mathbf{i}} \times F\hat{\mathbf{k}} + B\hat{\mathbf{j}} \times D\hat{\mathbf{i}} + B\hat{\mathbf{j}} \times E\hat{\mathbf{j}} + B\hat{\mathbf{j}} \times F\hat{\mathbf{k}} + C\hat{\mathbf{k}} \times D\hat{\mathbf{i}} + C\hat{\mathbf{k}} \times E\hat{\mathbf{j}} + C\hat{\mathbf{k}} \times F\hat{\mathbf{k}}$

$= A*D*\sin(0°) + A*E*\sin(90°)\hat{\mathbf{k}} - A*F*\sin(90°)\hat{\mathbf{j}} - B*D*\sin(90°)\hat{\mathbf{k}} + B*E*\sin(0°) + B*F*\sin(90°)\hat{\mathbf{i}} + C*D*\sin(90°)\hat{\mathbf{j}} - C*E*\sin(90°)\hat{\mathbf{i}} + C*F*\sin(0°)$

$= A*E\hat{\mathbf{k}} - A*F\hat{\mathbf{j}} - B*D\hat{\mathbf{k}} + B*F\hat{\mathbf{i}} + C*D\hat{\mathbf{j}} - C*E\hat{\mathbf{i}}$

$=$ **$(B*F - C*E)\hat{\imath} + (C*D - A*F)\hat{\jmath} + (A*E - B*D)\hat{k}$**