The Newton-Raphson method.

CS-116B: Computer Graphics Algorithms

Newton-Raphson method

- Newton-Raphson: also known as Newton's method.
- "...a root-finding algorithm that uses the first few terms of the Taylor series of a function f(x) in the vicinity of a suspected root."

Source: Source: Wolfram MathWorld at http://mathworld.wolfram.com/NewtonsMethod.html

Newton-Raphson method

- “This method starts at a guess for the unknown v^{t+1} and iteratively improves this guess.”
- “Newton’s first law says that without forces, velocities do not change which makes this guess a good one.”
- “The equations are linearized at the current state and the resulting linear system is solved to find a better approximation. This process is repeated until the error falls below a certain threshold.”

Source: *Real Time Physics Class Notes*, p. 17

Newton-Raphson method: Example

Solve: $x^3 - 2 * x = 0$

$f(x) = x^3 - 2 * x$

$f'(x) = 3*x^2 - 2$

$$x_{n+1} = x_n - f(x_n) / f'(x_n)$$

$$= x_n - (x^3 - 2*x) / (3*x^2 - 2)$$

Newton-Raphson method: Example

Step 1: $x_0 = 2$

$x_1 = x_0 – (x_0^3 - 2*x_0) / (3*x_0^2 – 2)$

$= 2 – (2^3 – 2* 2) / (3 * 2^2 – 2)$

$= 2 – (8 – 4) / (3 * 4 – 2)$

$= 2 – 4 / 10$

$= 2 – 0.4$

$= 1.6$

Newton-Raphson method: Example

Step 2: $x_1 = 1.6$

$x_2 = x_1 – (x_1^3 - 2*x_1) / (3*x_1^2 – 2)$

$= 1.6 – (1.6^3 - 2*1.6) / (3*1.6^2 – 2)$

$= 1.6 – (0.896 / 5.68)$

≈ 1.44225

Newton-Raphson method: Example

Step 3: $x_2 = 1.44225$

$x_3 = x_2 – (x_2^3 - 2*x_2) / (3*x_2^2 – 2)$

$\approx 1.44225 – (0.0272429)$

≈ 1.41501

Newton-Raphson method: Example

Step 4: $x_3 = 1.41501$

$x_4 \;= x_3 – (x_3^3 - 2*x_3) / (3*x_3^2 – 2)$

$\approx 1.41421 – (0.000796401)$

≈ 1.41421

Newton-Raphson method: Example

Step 5: $x_4 = 1.41421$

$x_5 = x_4 – (x_4^3 - 2*x_4) / (3*x_4^2 – 2)$

$\approx 1.41421 – (6.72981 \times 10^{-7})$

≈ 1.41421

Newton-Raphson method: Example

Step 6: $x_5 = 1.41421$

$x_6 \;\; = x_5 – (x_5^3 - 2*x_5) / (3*x_5^2 – 2)$

$\approx 1.41421 – (4.80394 \times 10^{-13})$

≈ 1.41421

Newton-Raphson method: Example

Solve: $x^3 - 2 * x = 0$

$x = 1.41421$

$1.41421^3 - 2 * 1.41421 = -0.000014249438539$